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A Leontief-type
Model of
Ownership
Structures
Methodology and
Implications
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Abstract

A simple algebraic model of a property structure leading to Leontief's input-output scheme is developed and used to eliminate indirect ownership relations and evaluate the final distribution of national property among individual owners. A concept of transparency of an ownership structure is defined. Implications of non-transparency for general equilibrium theory, profit distribution and decision making are discussed.

Keywords: ownership structure, primary owners, privatization illusion, secondary owners, transparency

JEL classification: C60, L33, K11
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1 Introduction

One of the basic paradigms of neo-classical economics reflected in general equilibrium theory and welfare economics is the assumption about the economic organization of the society based on private ownership of production factors and services and their use to maximize 'selfish' benefits of owners. Individuals as consumers are maximizing utility subject to budget constraint having on the right-hand side incomes from selling production factors and services owned by them and the revenues from profits of firms they are co-owning; firms are maximizing profits and the invisible hand of competition leads to Pareto optimal equilibrium states (Arrow, 1951, Debreu, 1959, Feldman, 1989).

Facing reality one can observe a quite different picture: a universe of corporations and non-transparent networks of ownership relations. Citizens are owners of a fraction of shares; ownership is dominated by big anonymous companies, banks and funds, who are co-owning a significant part of national property on an institutional basis. Citizen A has a share in corporation B, corporation B has a share in corporation C, corporation C has a share in corporation D, and corporation D has a share in corporation B. Is there some relation between citizen A and corporation D?

The legitimate question is: Can an anonymous institution as an institution own anything? Because of transaction costs modern economy cannot be governed by individual owners directly, so the system of agents has been developed, consisting of intermediary institutions and their professional management, mostly distinct from owners. But in principle intermediary institutions are only authorized to execute some of the property rights as agents and on behalf and for the benefit of individual owners. The final owners of national property can only be individuals or their non-profit associations.¹

Accepting this point of view, one can ask a rather technical question: In a non-transparent network of ownership relations, is there a possibility to disclose a final assignment of the whole national property to individual owners only? In the present paper we try to answer this question.

¹ "Property rights are of course human rights, i.e., rights which are possessed by human beings. The introduction of the wholly false distinction between property rights and human rights in many policy discussions is surely one of the all time great semantic flimflams" (Jensen and Meckling, 1976).
A simple algebraic model of ownership structures is formulated reflecting direct and indirect ownership relations. An iterative process of eliminating indirect relations is proposed. It is shown that this process converges to the ownership structure in which all intermediary indirect relations are eliminated and the property is fully attributed to individual owners. The concept of transparency of a given observable ownership structure is introduced: an ownership structure is called transparent if the iterative process leads to elimination of indirect relations in a finite number of iterations, otherwise the structure is considered not to be transparent. And finally, it is shown that using a Leontief-type model it is possible to evaluate the final distribution of property exactly (not as an approximation) even in the case of non-transparent ownership structures.

The idea of transparency based on convergence properties of an iterative process of indirect relations elimination, was proposed in Turnovec (1999). The new contribution presented in this paper is an extension of the Leontief input-output methodology on structural analysis of ownership relations.

2 **Model of ownership structures**

Let us consider two types of economic agents: the primary owners, who can own, but cannot be owned (citizens, citizens' non-profit associations, state, municipalities etc.), and the secondary owners, who can be owned and at the same time can own (companies, corporations).

Let

\[ m \] be the number of primary owners, \( i = 1, 2, \ldots, m \),
\[ n \] be the number of secondary owners (companies), \( j = 1, 2, \ldots, n \),
\[ s_{ji}^0 \] be the direct share of the primary owner \( i \) in the secondary owner \( j \) (as a proportion of the total number of shares),
\[ t_{jk}^0 \] be the direct share of the secondary owner (company) \( k \) in the secondary owner (company) \( j \).

Then the \( n \times m \) matrix

\[ S_0 = (s_{ji}^0) \]

where the row \( j \) expresses the shares of the primary owners \( i = 1, 2, \ldots, m \) in the secondary owner \( j \), and the column \( i \) expresses the shares of the primary owner \( i \) in the secondary

\[ ^2 \text{Speaking about direct relation we have in mind relation between individual A and company B providing that individual A owns a share in company B, while indirect relation means that individual A, having a share in company B and not having a share in company C, has through company B a relation to company C that is co-owned by company B.} \]
owners \( j = 1, 2, ..., n \), will be called a matrix of primary property distribution, and the \( n \times n \) matrix

\[
T_0 = (t^0_{jk})
\]

where the row \( j \) expresses the shares of the secondary owners \( k = 1, 2, ..., n \) in the secondary owner \( j \), and the column \( k \) expresses the shares of the secondary owner \( k \) in the secondary owners \( j = 1, 2, ..., n \), will be called a matrix of secondary property distribution. The couple

\[
(S_0, T_0)
\]

characterizes the initial property distribution in an economy.

Clearly

\[
\sum_{i=1}^{m} s^0_{ij} + \sum_{k=1}^{n} t^0_{jk} = 1
\]

for any \( j = 1, 2, ..., n \).

**Example 1**

*Let us consider a hypothetical initial ownership structure with the three primary owners \( P_1, P_2, P_3 \), and the three companies \( C_1, C_2, C_3 \) (secondary owners), described in Table 1.*

<table>
<thead>
<tr>
<th></th>
<th>Matrix ( S_0 )</th>
<th></th>
<th>Matrix ( T_0 )</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_1 )</td>
<td>( P_1 ) / 0.4</td>
<td>( P_2 ) / 0.2</td>
<td>( P_3 ) / 0.1</td>
<td>( C_1 ) / 0</td>
</tr>
<tr>
<td>( C_2 )</td>
<td>( P_1 ) / 0.55</td>
<td>( P_2 ) / 0.25</td>
<td>( P_3 ) / 0</td>
<td>( C_1 ) / 0.2</td>
</tr>
<tr>
<td>( C_3 )</td>
<td>( P_1 ) / 0.3</td>
<td>( P_2 ) / 0.3</td>
<td>( P_3 ) / 0.2</td>
<td>( C_1 ) / 0.1</td>
</tr>
</tbody>
</table>

*In this case*

\[
S_0 = \begin{pmatrix} 0.4 & 0.2 & 0.1 \\ 0.55 & 0.25 & 0 \\ 0.3 & 0.3 & 0.2 \end{pmatrix}
\]

and
Matrices $S_0$ and $T_0$ provide an observable property distribution.

If $T_0 = 0_{nn}$, where $0_{nn}$ is the nxn zero matrix, we have a very simple and transparent structure, when only primary owners own companies and there exists no indirect ownership.

However, in real economies we do not have such transparent structures, and that can lead to situations when it is not so easy to see who owns what. If a primary owner $A$ has a share in a secondary owner $B$, the secondary owner $B$ has a share in a secondary owner $C$, and the secondary owner $C$ has a share in a secondary owner $D$, then there exist direct ownership relations between $A$ and $B$ and $B$ and $C$, and indirect ownership relations between $A$ and $C$, $A$ and $D$ and $B$ and $D$. If moreover $D$ has a share in $B$, then the situation is completely unclear. The problem is how to evaluate direct and indirect ownership relations, and to identify the part of company $C$ which is owned by primary owner $A$ etc.

Assuming $T_0 \neq 0_{nn}$ let us consider a primary owner $i$. Clearly, his total share in the company (secondary owner) $j$ is given not only by his direct share $s_{0ji}$ in $j$, but also by the indirect share following from his shares in other secondary owners that are co-owning secondary owner $j$. This can be expressed as

$$s_j^i = s^0_{ji} + \sum_{r=1}^n t^0_{jr} s^0_{ri}$$

Consider a secondary owner $k$. His effective share in the company $j$ is given by the appropriate fractions of the shares that follow from his shares in other companies that co-own company $j$:

$$t_j^k = \sum_{r=1}^n t^0_{jr} t^0_{rk}$$

In matrix form we have

$$S_i = S_0 + T_0 S_0$$

$$T_1 = T_0 T_0 = T_0^2$$

So, considering indirect relations, we can obtain a decomposition of property on a direct component (following from registered shares of primary owners) and an indirect

$$T_0 = \begin{pmatrix} 0 & 0.3 & 0 \\ 0.2 & 0 & 0 \\ 0.1 & 0.1 & 0 \end{pmatrix}.$$
component (following from indirect relations). We shall call the initial distribution \((S_0, T_0)\) a distribution of zero degree, and the distribution \((S_1, T_1)\) a distribution of the first degree.

**Example 2**

In the ownership structure of Table 1 the matrix of secondary owners’ shares is non-zero, so there exist indirect ownership relations. Taking into account indirect relations, we obtain a more precise distribution:

\[
S_1 = \begin{pmatrix}
0.4 & 0.2 & 0.1 \\
0.55 & 0.25 & 0 \\
0.3 & 0.3 & 0.2
\end{pmatrix} + \begin{pmatrix}
0 & 0.3 & 0 \\
0.2 & 0 & 0 \\
0.1 & 0.1 & 0
\end{pmatrix} = \begin{pmatrix}
0.4 & 0.3 & 0.1 \\
0.55 & 0.25 & 0 \\
0.3 & 0.3 & 0.2
\end{pmatrix}
\]

\[
T_1 = \begin{pmatrix}
0.4 & 0.2 & 0.1 \\
0.55 & 0.25 & 0 \\
0.3 & 0.3 & 0.2
\end{pmatrix} + \begin{pmatrix}
0.165 & 0.075 & 0 \\
0.08 & 0.04 & 0.02 \\
0.095 & 0.045 & 0.01
\end{pmatrix} = \begin{pmatrix}
0.565 & 0.275 & 0.1 \\
0.63 & 0.29 & 0.02 \\
0.395 & 0.345 & 0.21
\end{pmatrix}
\]

and

\[
T_1 = \begin{pmatrix}
0 & 0.3 & 0 \\
0.2 & 0 & 0 \\
0.1 & 0.1 & 0
\end{pmatrix} + \begin{pmatrix}
0 & 0.3 & 0 \\
0.2 & 0 & 0 \\
0.1 & 0.1 & 0
\end{pmatrix} = \begin{pmatrix}
0.06 & 0.06 & 0 \\
0.02 & 0.03 & 0
\end{pmatrix}
\]

The recalculated distribution is set out in Table 2.

<table>
<thead>
<tr>
<th></th>
<th>Matrix (S_1)</th>
<th></th>
<th>Matrix (T_1)</th>
<th></th>
<th></th>
<th></th>
<th>total</th>
</tr>
</thead>
<tbody>
<tr>
<td>(C_1)</td>
<td>0.565</td>
<td>0.275</td>
<td>0.1</td>
<td>0.06</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>(C_2)</td>
<td>0.63</td>
<td>0.29</td>
<td>0.02</td>
<td>0</td>
<td>0.06</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>(C_3)</td>
<td>0.395</td>
<td>0.345</td>
<td>0.21</td>
<td>0.02</td>
<td>0.03</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Now we have a new distribution \((S_1, T_1)\) taking into account indirect relations. Matrix \(T_1\) is non-zero, so we have not disclosed the final distribution of property among the primary owners.
We can repeat all our considerations to produce a distribution of the second degree as

\[ S_2 = S_1 + T_1 S_1 = (S_0 + T_0 S_0) + T_0^2 (S_0 + T_0 S_0) = \]

\[ = (I + T_0 + T_0^2 + T_0^3) S_0 \]

\[ T_2 = T_1 T_1 = T_0^4 \]

e tc.

In the general case

\[ S_r = S_{r-1} + T_{r-1} S_{r-1} \]

\[ T_r = T_{r-1} T_{r-1} \]

\((r = 1, 2, ..., k, ...)\), or

\[ S_r = \left( I + \sum_{j=1}^{r-1} T_0^j \right) S_0 \]

\[ T_r = T_0^r \]

To eliminate indirect relations there should exist a positive integer \( r \) such that

\[ T_r = T_0^r = 0_{nn} \]

**Example 3**

*In Table 3 we have the next iteration of our eliminating process. We can still observe some residual indirect property relations.*

<table>
<thead>
<tr>
<th></th>
<th><strong>Matrix S₂</strong></th>
<th></th>
<th><strong>Matrix T₂</strong></th>
<th></th>
<th>total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( P_1 )</td>
<td>( P_2 )</td>
<td>( P_3 )</td>
<td>( C_1 )</td>
<td>( C_2 )</td>
</tr>
<tr>
<td>( C_1 )</td>
<td>0.5989</td>
<td>0.2915</td>
<td>0.106</td>
<td>0.0036</td>
<td>0</td>
</tr>
<tr>
<td>( C_2 )</td>
<td>0.6678</td>
<td>0.3074</td>
<td>0.0212</td>
<td>0</td>
<td>0.0036</td>
</tr>
<tr>
<td>( C_3 )</td>
<td>0.4252</td>
<td>0.3592</td>
<td>0.2126</td>
<td>0.00122</td>
<td>0.0018</td>
</tr>
</tbody>
</table>
In fact we state that in this particular case we shall never be able to find the final assignment of property to the primary owners.

Let us consider now the simple structure in Table 4.

Table 4

<table>
<thead>
<tr>
<th></th>
<th>Matrix $S_0$</th>
<th></th>
<th>Matrix $T_0$</th>
<th></th>
<th></th>
<th>total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$P_1$</td>
<td>$P_2$</td>
<td>$P_3$</td>
<td>$C_1$</td>
<td>$C_2$</td>
<td>$C_3$</td>
</tr>
<tr>
<td>$C_1$</td>
<td>0.6</td>
<td>0.2</td>
<td>0.1</td>
<td>0</td>
<td>0.1</td>
<td>0</td>
</tr>
<tr>
<td>$C_2$</td>
<td>0.4</td>
<td>0.1</td>
<td>0.2</td>
<td>0</td>
<td>0</td>
<td>0.3</td>
</tr>
<tr>
<td>$C_3$</td>
<td>0.3</td>
<td>0.3</td>
<td>0.4</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

In Table 5 we have the result of the second iteration. In this particular case we succeeded in eliminating indirect relations and identifying the final distribution of property among the primary owners.

Table 5

<table>
<thead>
<tr>
<th></th>
<th>Matrix $S_2$</th>
<th></th>
<th>Matrix $T_2$</th>
<th></th>
<th></th>
<th>total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$P_1$</td>
<td>$P_2$</td>
<td>$P_3$</td>
<td>$C_1$</td>
<td>$C_2$</td>
<td>$C_3$</td>
</tr>
<tr>
<td>$C_1$</td>
<td>0.649</td>
<td>0.219</td>
<td>0.132</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$C_2$</td>
<td>0.490</td>
<td>0.190</td>
<td>0.320</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$C_3$</td>
<td>0.300</td>
<td>0.300</td>
<td>0.400</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

3 Transparency

Intuitively, the concept of transparency of a property structure should be related to the possibility to eliminate indirect relations and to find the final assignment of the total property to primary owners only.

Within the framework of the model described above, the sequence of matrices $T_0, T_1, T_2, \ldots$ can be used for quantification of the concept of transparency of property distribution.

If we accept as an axiom that finally any distribution of property is distribution among the primary owners only, then transparency of a particular initial distribution can be measured by the distance of the primary distribution from the final distribution taking into account all degrees of indirect links.

The maximum of transparency is achieved when $T_0 = 0$. In this case primary distribution is transparent in the sense that any property is related to primary owners only and no indirect relations appear.
We shall say that a particular property structure \((S_0, T_0)\) such that \(T_0 \neq 0_{nn}\) is k-transparent, if in property distribution \((S_k, T_k)\) of degree k it holds that \(T_k = 0_{nn}\), while in property distribution of degree k-1 \((S_{k-1}, T_{k-1})\) it holds that \(T_{k-1} \neq 0_{nn}\).

A property structure is non-transparent, if for any positive integer k it holds that \(T_k \neq 0_{nn}\).

**Lemma 1**

Let \(A\) be a square \(n \times n\) matrix such that the sequence

\[
A^1, A^2, \ldots, A^k, \ldots
\]

of powers of the matrix \(A\) converges to a zero matrix, i.e.

\[
\lim_{k \to \infty} A^k = 0_{nn}
\]

Then

1. either there exists a positive integer \(s \leq n\) such that

\[
A^{s-1} \neq 0_{nn} \text{ and } A^s = 0_{nn}
\]

or

\[
A^k \neq 0_{nn}
\]

for any positive integer \(k\).

2. Matrix \(I - A\) is non-singular (\(I\) being an \(n \times n\) identity matrix) and

\[
(I - A)^{-1} = \sum_{k=0}^{\infty} A^k
\]

PROOF of the first part is based on the properties of so-called nilpotent matrices (e.g. Archibald, 1968), of the second part on Leontief (1966).

The matrix \(T_0\) of order \(n\) is assumed to be such that

\[
\sum_{k=1}^{n} \gamma_{jk}^0 < 1
\]

Then clearly the sequence of its powers converges to a zero matrix and the conditions of the theorem are satisfied. From Lemma 1 it follows that either there exists \(k\) such that 2 \(k \leq n\), \(T_k \neq 0_{nn}\) and \(T_{k+1} = 0_{nn}\), or \(T_r \neq 0_{nn}\) for any integer \(r\).
Example 4

Let us consider matrix $T_0$ from the first property structure:

$$
T_0 = \begin{pmatrix}
0 & 0.3 & 0 \\
0.2 & 0 & 0 \\
0.1 & 0.1 & 0
\end{pmatrix}
$$

Then

$$
T_1 = \begin{pmatrix}
0.2 & 0 & 0 \\
0.1 & 0.1 & 0
\end{pmatrix}
$$

and

$$
T_2 = T_0^2 = \begin{pmatrix}
0.0036 & 0 & 0 \\
0 & 0.0036 & 0 \\
0.0012 & 0.0018 & 0
\end{pmatrix}
$$

In our case $n = 3$, for $k = 1$ we have $2^1 = 2 < 3$, $T_1 \neq 0_{n \times n}$, and for $k + 1$ we have $T_2 \neq 0_{n \times n}$, where $2^2 = 4 > 3$, hence the alternative b) of the theorem statement appears. The property structure is not transparent.

4 One interesting identity

Let us assume that there exists a final distribution of property among primary owners without any indirect links. Let $x_{ji}$ be the full (direct and indirect) share of primary owner $i$ in corporation $j$. Let us call the $n \times m$ matrix $X = (x_{ji})$ a matrix of final distribution. In case of a transparent ownership structure we know that

$$
X = S_t = \left( I + \sum_{j=1}^{2^1 - 1} T_0^j \right) S_0
$$

where $2^1 - 1 \leq n$, $n$ is the number of secondary owners.

Question: Is it possible to evaluate exactly the matrix $X$ also in the case when the initial ownership structure is not transparent?

Lemma 2

Let $(S_0, T_0)$ be the initial ownership structure such that
\[ s^{0}_{ji} \geq 0, \quad t^{0}_{jk} \geq 0 \]

and

\[ \sum_{i=1}^{m} s^{0}_{ji} + \sum_{k=1}^{n} t^{0}_{jk} = 1 \]

and let there exist a non-negative integer r such that

\[ \sum_{k=1}^{n} t^{0}_{jk} < 1 \]

for all \( j = 1, 2, ..., n \). Then the sequence \( S_r \) converges and

\[ \lim_{r \to \infty} S_r = (\sum_{i=0}^{\infty} T_r) S_0 = (1 - T_0)^3 S_0 = X \]

We have obtained an identity that is well known from Leontief’s input–output models.

### 5 The privatization illusion

Using the structural approach described above, we can try to answer the question: How much privatized is an ‘almost fully’ privatized economy?

Let \( w_j \) be the weight of a company \( j \) (e.g., the market value, value of assets etc.). Considering a distribution \( (S_r, T_r) \) of degree \( r \), we can evaluate the corresponding distribution of the total property in an economy as

\[ p^r_i = \frac{\sum_{j=1}^{n} s^r_{ij} w_j}{\sum_{j=1}^{n} w_j}, \quad d^r_k = \frac{\sum_{j=1}^{n} t^r_{kj} w_j}{\sum_{j=1}^{n} w_j} \]

where \( p^r_i \) is the share of the \( i \)-th primary owner and \( d^r_k \) is the share of the \( k \)-th company (secondary owner) in the total property according to distribution of degree \( r \). Let us illustrate by a simple example that the primary distribution of national property can significantly differ from the final distribution reflecting indirect links.

**Example 5**

Let us assume that an economy consists of the following five actors: the state \( S \), group of individual investors \( M \), two banks \( B1 \) and \( B2 \), investment fund \( F \) and a group of industrial
enterprises I. In Table 6 we provide a hypothetical primary property distribution in such an economy.

<table>
<thead>
<tr>
<th></th>
<th>S</th>
<th>M</th>
<th>B1</th>
<th>B2</th>
<th>F</th>
<th>I</th>
<th>total</th>
<th>weights</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>0.6</td>
<td>0.3</td>
<td>0</td>
<td>0</td>
<td>0.1</td>
<td>0</td>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>B2</td>
<td>0.7</td>
<td>0.2</td>
<td>0.1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>F</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>35</td>
</tr>
<tr>
<td>I</td>
<td>0</td>
<td>0.3</td>
<td>0</td>
<td>0</td>
<td>0.7</td>
<td>0</td>
<td>1</td>
<td>50</td>
</tr>
<tr>
<td>total share</td>
<td>0.095</td>
<td>0.19</td>
<td>0.005</td>
<td>0.35</td>
<td>0.36</td>
<td>0</td>
<td>1</td>
<td>50</td>
</tr>
</tbody>
</table>

We can see that, with respect to the initial property distribution, the total share of the state in the national property is 9.5%.

Table 7 indicates the property distribution of degree 1, taking into account indirect relations.

<table>
<thead>
<tr>
<th></th>
<th>S</th>
<th>M</th>
<th>B1</th>
<th>B2</th>
<th>F</th>
<th>I</th>
<th>total</th>
<th>weights</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>0.6</td>
<td>0.3</td>
<td>0</td>
<td>0.1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>B2</td>
<td>0.76</td>
<td>0.23</td>
<td>0</td>
<td>0</td>
<td>0.01</td>
<td>0</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>F</td>
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<td>0.36</td>
<td>0.0005</td>
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In this case, considering indirect links in cross-ownership, the share of the state has increased to 34.3%.

Table 8 presents the property distribution of degree 2, taking into account other indirect relations.

<table>
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<tr>
<th></th>
<th>S</th>
<th>M</th>
<th>B1</th>
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<th>F</th>
<th>I</th>
<th>total</th>
<th>weights</th>
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Comparing Table 6 and Table 8 we observe a significant increase in the share of the state in the national property: from 9.5% to 63.795%. And we still do not have the final distribution, assigning the shares to the primary owners only.

Let us use Lemma 2. In our particular case

\[
(I - T_0) = \begin{pmatrix}
1 & 0 & -0.1 & 0 \\
-0.1 & 1 & 0 & 0 \\
0 & -1 & 1 & 0 \\
0 & 0 & -0.7 & 1
\end{pmatrix}
\]

and

\[
(I - T_0)^{-1} = \begin{pmatrix}
1.010101 & 0.10101 & 0.10101 & 0 \\
0.10101 & 1.010101 & 0.010101 & 0 \\
0.10101 & 1.010101 & 1.010101 & 0 \\
0.070707 & 0.707071 & 0.707071 & 1
\end{pmatrix}
\]

Then

\[
X = (I - T_0)^{-1}S_0 =
\begin{pmatrix}
1.010101 & 0.10101 & 0.10101 & 0 \\
0.10101 & 1.010101 & 0.010101 & 0 \\
0.10101 & 1.010101 & 1.010101 & 0 \\
0.070707 & 0.707071 & 0.707071 & 1
\end{pmatrix}
\begin{pmatrix}
0.6 \\
0.7 \\
0 \\
0
\end{pmatrix}

= \begin{pmatrix}
0.676768 \\
0.767677 \\
0.767677 \\
0.537374
\end{pmatrix}
\begin{pmatrix}
0.323232 \\
0.232323 \\
0.232323 \\
0.462626
\end{pmatrix}
\]

and the final distribution of shares, after elimination of indirect links, will look as follows:
Table 9

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<th>S</th>
<th>M</th>
<th>B1</th>
<th>B2</th>
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<th>total</th>
<th>weights</th>
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6 An application: the Czech banking sector

In this part we demonstrate the possibility of practical implementation of our model on an analysis of the property structure of the core banking sector in the Czech Republic at the end of 1997. There were five major banks, representing almost 90% of the total assets of the Czech banking sector (Matoušek, 1998):
- CS Česká spořitelna (Czech Saving Bank),
- CP Česká pojišťovna (Czech Insurance),
- KB Komerční banka (Commercial Bank),
- IPB Investiční a poštovní banka (Investment and Post bank),
- CSOB Československá obchodní banka (Czecho-Slovak Trade Bank).

As primary owners we have:
- FNM Fond národního majetku (Fund of National Property), state agency,
- CNB Česká národní banka (Czech National Bank), central bank,
- MF Ministerstvo financí (Ministry of Finance), state agency,
  Mun. Sdružení měst (Association of Municipalities),
- BH Bank Holding, non-state,
- JRING J. Ring stock comp., non-state,
- PPF I First Privatization Holding, non-state,
- BNY The Bank of New York,
- Nomura Nomura Group,
- MB The Midland Bank,
- BTI The Bankers Trust Investment,
- SR Slovak Republic,
- others minority investors (mostly from voucher privatization).
The secondary owners are:

- SPIF-C Spořitelní privatizační investiční fond – Český (investment fund),
- SPIF-V Spořitelní privatizační investiční fond – výnosový (investment fund),
- PPF První privatizační fond (investment fund),
- PIF První investiční fond (investment fund),
- RIF Restituční investiční fond (investment fund),
- IPF-K Investiční privatizační fond banky (investment fund),
- VS Vojenské stavby (stock company).

The structure is incomplete, because some of our primary owners are in fact secondary owners as well (owned mostly by foreign capital), but to have a closed system for illustrative purposes, we shall not go any deeper.

Table 10 gives the initial ownership distribution (end of 1997). In Table 15 we obtained the final ownership distribution after elimination of indirect relations. We provide also intermediate calculations. We can see, for example, that the difference between final and initial distribution can mean the difference between majority control and minority (Komerční banka). While the matrix $S_0$ of initial distribution is pretty sparse, the matrix $X$ of final distribution introduces new additional fractions of property to all primary owners.
Table 10

Initial property distribution in the banking sector of the Czech Republic, end of 1997, in relative shares

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<th>JRING</th>
<th>PPF</th>
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<th>Nomura</th>
<th>MB</th>
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15
Table 11

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Table 12

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Matrix $X = (I - T_0)^{-1}S_0$

Final property distribution after elimination of indirect relations

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Table 15
7 Some implications

There can be a significant difference between the primary ‘face’ image of the ownership structure and the ‘true’ position of the subjects of property rights. This difference has serious theoretical implications.

Just a few questions:

(a) Profits: How are, and how should they be, distributed? We established that the final allocation of property to the individual property owners, after elimination of indirect relations, is

\[ X = (I - S_0)^{-1} S_0 \]

while only the listed direct initial distribution \( S_0 \) is taken into account.

(b) Decision making power: How is, and how should it be, distributed? According to \( X \) or according to \( S_0 \)\(^3\)

(c) Another issue for theoretical research is the implication of non-transparency of ownership structures for the general equilibrium and welfare theory. Indirect ownership relations clearly generate externalities in the profit maximization doctrine of general equilibrium theory: total profit of one company might depend on profits of other companies.

Many problems associated with the inadequacy of the current general equilibrium theory and welfare economics can be related to the theory of agency relationships (principal–agent problem). An agency relationship is a contract under which one or more persons (the principal(s)) engage another person (the agent) to perform some service on their behalf which involves delegating some decision making authority to the agent and providing some incentive scheme for the agent to maximize the welfare of the principal. Agency relations have been intensively investigated at firm level (see e.g. Jensen and Meckling, 1976, Varian, 1992). But here we face the overall economy level of the principal–agent problem. Indirect ownership relations, generally viewed as full ownership relations, are frequently just agency relations. We are living in an economy of agents behaving as owners. There is a hierarchical structure of agents in the economy. Primary owners are principals and secondary institutional owners are in many cases just labels for agents. But in the network of indirect ownership relations an agent A becomes a principal with respect to some other agent B, the agent B becomes a principal with respect to some other agent C, and C can become a principal with respect to A, principal of his principals. So finally it is not clear who

\(^3\) An agenda for future research is to apply the methodology developed here to the control structures that are given not by shares only, but by voting majorities, coalitions of owners etc. (see Maeland, 1991, Gambarelli, 1994).
is an agent and who is his principal. Such a situation can be considered a market imperfection and can lead to market failures.\footnote{Not understanding clearly distinction between principals and agents and absence of agency relation regulation was one of the reasons of problematic results of the Czech privatization (see Bohatá, 1998, Schwartz, 1997).}

A hierarchical principal–agent problem within the framework of general equilibrium theory and welfare economics is a challenge for economic theory.
References


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